GCSE Mathematics Foundation Tier

Number · Algebra · Ratio, proportion and rates of change · Geometry & measures · Probability · Statistics

Here is pretty much all the Foundation Tier content we could fit onto an A3 sheet of paper, including all the formulae you are required to know for GCSE. An ➔ points to an illustrative example. The codes refer to the DfE subject content. Pin this to a wall, keep it on your desk, carry it in your bag, make notes on it (sorry, don't take it into the examination)…

BIDMAS N3

…or BODMAS. Use the correct order of operations; take care when using a calculator.

- **B**rackets
- **I**ndices (or p**O**wers)
- **D**ivision and **M**ultiplication
- **A**ddition and **S**ubtraction

Types of number N4

Integer: a "whole" number
Factors; the divisors of an integer
➔ **Factors of 12 are 1, 2, 3 ,4, 6, 12**
Multiples; a "times table" for an integer (will continue indefinitely)
➔ **Multiples of 12 are 12, 24, 36 …**
Prime number: an integer which has exactly two factors (1 and the number itself). Note: 1 is not a prime number.

HCF, LCM N4

Highest **C**ommon **F**actor (HCF)
➔ **Factors of 6 are 1, 2, 3, 6**
Factors of 9 are 1, 3, 9
HCF of 6 and 9 is 3
Lowest **C**ommon **M**ultiple (LCM)
➔ **Multiples of 6 are 6, 12, 18, 24, …**
Multiples of 9 are 9, 18, 27, 36, …
LCM of 6 and 9 is 18

Prime factors N4

Write a number as a product of its prime factors; use indices for repeated factors:
➔ $720 = 5 \times 3^2 \times 2^4$

Powers and roots N6, N7

Special indices: for any value a:

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

➔ $3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

Calculating with fractions N8

Adding or subtracting fractions; use a common denominator…
➔ $\frac{4}{5} - \frac{1}{3} = \frac{12}{15} - \frac{5}{15} = \frac{7}{15}$
Multiplying fractions; multiply numerators and denominators…
➔ $\frac{4}{7} \times \frac{2}{3} = \frac{8}{21}$
Dividing fractions; "flip" the second fraction, then multiply…
➔ $\frac{2}{7} \div \frac{5}{6} = \frac{2}{7} \times \frac{6}{5} = \frac{12}{35}$

Fractions, decimals N10

Fraction is numerator ÷ denominator
➔ $\frac{5}{8} = 5 \div 8 = 0.625$
Use place values to change decimals to fractions. Simplify where possible.
➔ $0.45 = \frac{45}{100} = \frac{9}{20}$
Learn the most frequently used ones:

$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{4}$
0.5	0.25	0.1	0.2	0.75

Surds N8

Look for the biggest square number factor of the number:
➔ $\sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5}$

Standard form N9

Standard form numbers are of the form $a \times 10^n$, where $1 \le a < 10$ and n is an integer.

Standard units N13

1 tonne = 1 000 kilograms
1 kilogram = 1 000 grams

1 kilometre = 1 000 metres
1 metre = 100 centimetres
= 1 000 millimetres
1 centimetre = 10 millimetres

1 day = 24 hours
1 hour = 60 minutes = 3 600 seconds
1 minute = 60 seconds

Rounding N15

Truncate the number, then use a "decider digit" to round up or down.
Decimal places: use the decimal point
➔ **162.3681 to 2dp;**
162.36 | 81 **= 162.37 to 2dp**
Significant figures: use the first non-zero digit.
➔ **162.3681 to 2sf;**
16 | 2.3681 **= 160 to 2sf**
➔ **0.007 039 to 3sf;**
0.007 03 | 9 **= 0.007 04 to 3sf**

Error intervals N15

Find the range of numbers that will round to a given value:
➔ $x = 5.83$ **(2 decimal places)**
$5.825 \le x < 5.835$
➔ $y = 46$ **(2 significant figures)**
$45.5 \le y < 46.5$
Note use of ≤ and <, and that the last significant figure of each is 5.

Algebraic notation A1

$$ab = a \times b$$
$$3y = y + y + y$$
$$a^2 = a \times a$$
$$a^3 = a \times a \times a$$
$$a^2b = a \times a \times b$$
$$\frac{a}{b} = a \div b$$

Equations and identities A3

An equation is true for some particular value of x…
➔ $2x + 1 = 7$ **is true if** $x = 3$
…but an identity is true for every value of x
➔ $(x + a)^2 \equiv x^2 + 2ax + a^2$
(note the use of the symbol ≡)

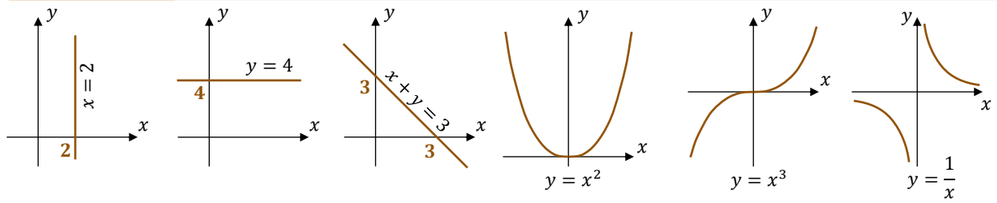
Laws of indices A4

For any value a:

$$a^x \times a^y = a^{x+y}$$
$$\frac{a^x}{a^y} = a^{x-y}$$
$$(a^x)^y = a^{xy}$$

➔ $\left(\frac{2pq^4}{p^3q}\right)^3 = \frac{8p^3q^{12}}{p^9q^3} = \frac{8q^9}{p^6}$ or $8q^9p^{-6}$

Standard graphs A12



$y = mx + c$ A9

Equation of straight line $y = mx + c$
m is the gradient; c is the y intercept:
➔ **Find the equation of the line that joins (0 , 3) to (2 , 11)**
Find its gradient…

$$\frac{11 - 3}{2 - 0} = \frac{8}{2} = 4$$

…and its y intercept…
Passes through (0 , 3), so c = 3.
Equation is $y = 4x + 3$.

Parallel lines: gradients are equal;
➔ $y = 2x + 3$ **and** $y = 2x - 5$ **both have gradient 2, so are parallel.**

Expanding brackets A4

$$p(q + r) = pq + pr$$
➔ $5(x - 2y) = 5x - 10y$
$$(x + a)(x + b) = x^2 + ax + bx + ab$$
➔ $(2x - 3)(x + 5)$
$= 2x^2 - 3x + 10x - 15$
$= 2x^2 + 7x - 15$
Reverse of expanding is factorising - putting an expression into brackets.

Quadratics A18

Solve a quadratic by factorising.
➔ **Solve** $x^2 - 8x + 15 = 0$
Put into brackets (taking care with any negative numbers)…
$(x - 3)(x - 5) = 0$
…then either $x - 3 = 0$ or $x - 5 = 0$, so that $x = 3$ **or** $x = 5$.

Difference of two squares A4

$$a^2 - b^2 = (a + b)(a - b)$$
➔ $x^2 - 25 = (x + 5)(x - 5)$

Simultaneous equations A19

➔ **Solve** $\begin{cases} 2x + 3y = 11 \\ 3x - 5y = 7 \end{cases}$
Multiply to match a term in x or y
$\begin{cases} 10x + 15y = 55 \\ 9x - 15y = 21 \end{cases}$
Add or subtract to cancel…
$19x = 76$, **so** $x = 4$
Finally, substitute and solve…
$2 \times 4 + 3y = 11$, **so** $y = 1$

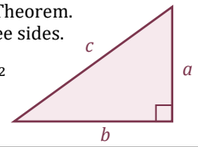
Rearrange a formula A5

The subject of a formula is the term on its own. Use rules that "balance" the formula to change its subject
➔ **Make** x **the subject of**
$2x + 3y = z$
Here, subtract $3y$ from both sides…
$2x = z - 3y$
…then divide both sides by 2…

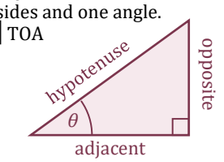
$$x = \frac{z - 3y}{2}$$

Right angled triangles G20, G22

Pythagoras Theorem.
Links all three sides.
No angles.
$a^2 + b^2 = c^2$



Trigonometry.
Links two sides and one angle.
SOH | CAH | TOA



$\sin\theta = \frac{\text{opp}}{\text{hyp}}$ $\cos\theta = \frac{\text{adj}}{\text{hyp}}$ $\tan\theta = \frac{\text{opp}}{\text{adj}}$

Use "2ndF" or "SHIFT" key to find a missing angle

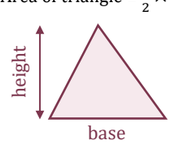
The longest side of any right angled triangle is the hypotenuse; check that your answer is consistent with this.

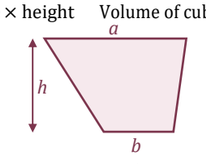
Special values of sin, cos, tan
Learn (or be able to find without a calculator)…

$\theta°$	$\sin\theta°$	$\cos\theta°$	$\tan\theta°$
0	0	1	1
30	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90	1	0	

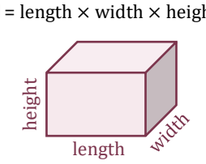
Areas and volumes G16, G17, G18, G23

Area of triangle = $\frac{1}{2} \times$ base $\times$ height



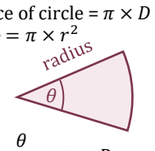


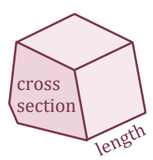
Volume of cuboid = length × width × height

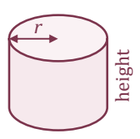


Area of trapezium = $\frac{1}{2}(a + b) \times h$

Circumference of circle = $\pi \times D$
Area of circle = $\pi \times r^2$







Arc length = $\frac{\theta}{360°} \times \pi \times D$

Area of sector = $\frac{\theta}{360°} \times \pi \times r^2$

Volume of cylinder = $\pi r^2 \times$ height

Volume of prism = area of cross section × length

Transformations G7, G8

Reflection
- Line of reflection

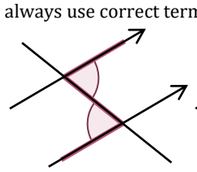
Translation
- Vector

Rotation
- Centre of rotation
- Angle of rotation
- Clockwise or anticlockwise

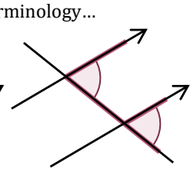
Enlargement
- Centre of enlargement
- Scale factor (if SF < 1 the shape will get smaller).

Angle facts G3

Equal angles in parallel lines: always use correct terminology…

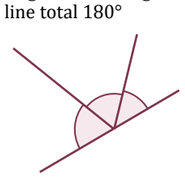


Alternate angles

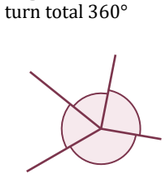


Corresponding angles

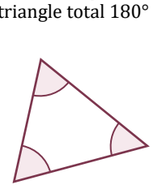
Angles on a straight line total 180°



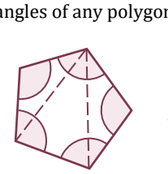
Angles in a full turn total 360°



Interior angles in a triangle total 180°

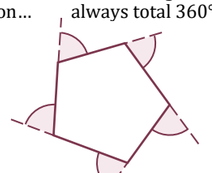


Use this for the interior angles of any polygon…



…or 180° × $(n - 2)$

Exterior angles always total 360°



Sequences A24, A25

Triangular numbers:

1st	2nd	3rd	4th	5th
1	3	6	10	15

Square numbers ($n^2 = n \times n$):

1^2	2^2	3^2	4^2	5^2
1	4	9	16	25

Cube numbers ($n^3 = n \times n \times n$):

1^3	2^3	3^3	4^3	5^3
1	8	27	64	125

nth term of an arithmetic (linear) sequence is $an + d$
➔ **nth term of 5,8,11,14,… is $3n+2$ (always increases by 3; first term is $3 \times 1 + 2 = 5$.)**
Geometric sequence; multiply each term by a constant ratio
➔ **3, 6, 12, 24, … (ratio is 2)**
Fibonacci sequence; make the next term by adding the previous two …
➔ **2, 4, 6, 10, 16, 26, 42, …**

Probability P8, P9

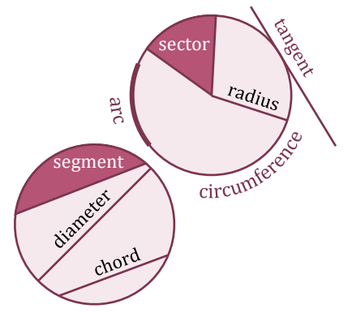
$$p = \frac{n(\text{equally likely favourable outcomes})}{n(\text{equally likely possible outcomes})}$$

$p = 0$	impossible
$0 < p < 0.5$	unlikely
$p = 0.5$	evens
$0.5 < p < 1$	likely
$p = 1$	certain

Probability rules P8, P9

Multiply for independent events
➔ **P(6 on dice and H on coin)**
$\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$
Add for mutually exclusive events
➔ **P(5 or 6 on dice)**
$\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$
Apply these rules to tree diagrams.

Parts of a circle G9



Division using ratio R5

Use a ratio for unequal sharing
➔ **Divide £480 in the ratio 7 : 5**
7 + 5 = 12, then £480 ÷ 12 = £40
7 × £40 = £280, 5 × £40 = £200
(check: £280 + £200 = £480 ✓)

Ratio and fractions R8

Link between ratios and fractions
➔ **Boys to girls in ratio 2 : 3**
$\frac{2}{5}$ are boys, $\frac{3}{5}$ are girls.

Percentages R9

$$y \text{ percent of } x = \frac{y}{100} \times x$$

➔ **Increase £58 by 26%.**
$\frac{26}{100} \times £58 = £15.08$
£58 + £15.08 = £73.08

$$y \text{ as a percentage of } x = \frac{y}{x} \times 100\%$$

➔ **The population of a town increases from 3 500 to 4 620. Find the percentage increase.**
$\frac{1\,120}{3\,500} \times 100\% = 32\%$
Note: fraction $= \frac{\text{increase}}{\text{original}}$
Learn the most frequently used ones:

$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{100}$
50%	25%	10%	20%	1%

Speed, distance, time R11

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

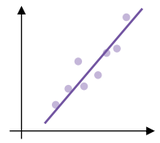
➔ **A car travels 90 miles in 1 hour, 30 minutes. Find its average speed.**
90 miles ÷ 1.5 hours = 60 mph

Averages S4

Mode: most frequently occurring
Median: put the data in numerical order, then choose the middle one
Mean $= \frac{\text{total of items of data}}{\text{number of items of data}}$

Correlation S6

Positive correlation



Negative correlation